Correlation of Local Velocities in Tubes, Annuli, and Parallel Plates

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A consistent method is presented for predicting local velocities in smooth tubes, concentric annuli, and parallel plates. Consideration is limited to the steady, isothermal, fully turbulent flow of constant-density fluids. Experimental data show the proposed correlation to be independent of Reynolds number and radius ratio. Intermediate quantities, calculated from friction data, permit local velocities to be determined over a wide range of operating conditions.

A number of experimental investigations have dealt with the distribution of mean local velocities in fully turbulent, isothermal flow through smooth tubes, but far fewer velocity data are available for flow in conduits of noncircular cross-section. Some recent publications have indicated that velocity profiles in concentric annuli and between parallel flat plates can be correlated with those in tubes by means of geometrical transformations. Although each case has been handled separately with more or less success, no more general approach has been forthcoming.

It is the purpose of this paper to present a consistent method of correlating mean local velocities in tubes, annuli, and parallel plates. Consideration is limited to steady, isothermal, fully turbulent flow of fluids having constant density. The recommended correlations are consistent with theoretical relationships developed for fully viscous, or laminar, flow, no attempt being made to deal with the region of laminar-turbulent transition. New velocity data have been obtained for flow between parallel plates and in one concentric annulus to aid in verifying the proposed correlations.

STATUS OF THE LITERATURE

Rothfus and Monrad (*8*) have shown that the Reynolds number effect on the u^+, y^+ correlation of local velocities in smooth tubes can be removed empirically, for practical purposes, by modification of the correlating parameters. They suggest the use of the dimensionless coordinates

$$U^+ = \frac{u}{\sqrt{\tau_0 g_0/\rho}}\left(\frac{V}{u_m}\right)_p \quad (1)$$

and

$$Y^+ = \frac{y\sqrt{\tau_0 g_0/\rho}}{\nu}\left(\frac{u_m}{V}\right)_p \quad (2)$$

It has been observed that U^+ is an essentially unique function of Y^+ over most of the stream in fully turbulent flow. Very close to the wall, the empiricism must break down since viscous behavior is approximated in that region. The modified u^+, y^+ parameters, however, make it possible to reduce turbulent velocity profiles in smooth tubes to a single functional relationship which is independent of Reynolds number to a satisfactory extent. Thus a starting point has been established from which to proceed in studying flow through noncircular ducts.

Rothfus and Monrad have noted that fully viscous velocity profiles in tubes and parallel plates are coincident whenever the radius of the tube equals the half-clearance between the plates, the skin frictions are made equal, and the same fluid flows in both conduits. Assuming coincident velocity profiles in fully turbulent flow under the same restraints, the authors predicted that

$$U_P^+ = U_p^+ \quad (3)$$

and

$$Y_P^+ = Y_p^+ \quad (4)$$

If the Reynolds number for parallel plates is defined on the basis of a hydraulic radius over the whole stream in the usual manner,

$$N_{Re_P} = \frac{4bV_P}{\nu} \quad (5)$$

The Reynolds number for tubes is ordinarily defined as

$$N_{Re_p} = \frac{2r_0 V_p}{\nu} \quad (6)$$

Since the maximum local velocity is the same for both types of ducts and $r_0 = b$, then it must follow that

$$N_{Re_p} = \frac{1}{2} N_{Re_P} \frac{(V/u_m)_p}{(V/u_m)_P} \quad (7)$$

if the local velocities are truly coincident as assumed. In like manner, since the skin frictions and fluid densities are equal,

$$\sqrt{\frac{f_p}{f_P}} = \frac{(V/u_m)_P}{(V/u_m)_p} \quad (8)$$

Thus it should be possible to predict ratios of average to maximum velocity from friction data or, conversely, friction factors from velocity ratios.

The data of Sage and coworkers (*1, 6, 7*) were shown to be in excellent agreement with Equations (3) and (4). The values of $(V/u_m)_p$ obtained from tube data (*5, 10, 11*) at the Reynolds number defined in Equation (6) altered the ordinary u^+, y^+ correlation for parallel plates in such fashion as to yield a correlation on U^+, Y^+ coordinates which was independent of Reynolds number and coincident with the unique curve for smooth tubes. Friction factors computed by means of Equation (8) were in satisfactory agreement with experimental friction data. The latter data deviated somewhat from the values predicted by means of the usual hydraulic radius concept. It is the opinion of the writers, however, that the deviation was probably less than the experimental uncertainty. Since infinitely broad parallel plates must be approximated experimentally by using a rectangular duct of large aspect ratio, it is almost impossible to correct for the effect of the side walls on the friction and over-all flow pattern.

Several years ago, Rothfus, Monrad, and Senecal (*9*) obtained measurements of fluid friction and velocity distribution in two smooth, concentric annuli. They observed that the radius of maximum local fluid velocity r_m attained the same value in fully turbulent flow as that predicted from theory for fully viscous flow; namely

$$r_m^2 = \frac{r_2^2 - r_1^2}{\ln\left(\frac{r_2}{r_1}\right)^2} \quad (9)$$

They also observed that the Fanning friction factor at the outer boundary of an annulus, defined by the equation

$$\tau_2 g_0 = \frac{f_2}{2}\rho V_a^2 \quad (10)$$

could be empirically correlated over the fully turbulent range in a manner independent of the radius ratio r_1/r_2. It was found that if the hydraulic radius concept were applied to the portion of the fluid lying between the radius of maximum velocity and the outer wall of the annular space, the friction factor f_2 could be obtained with excellent accuracy from the smooth-tube correlation at the Reynolds number

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$$N_{Re_e} = \frac{2(r_2^2 - r_m^2)V_a}{r_2\nu} \qquad (11)$$

The distribution of shearing stress in an annulus is not linear in the distance r from the center of the configuration, nor is the viscous velocity distribution parabolic in that distance. Through study of the theoretical velocity profile for wholly viscous motion (4), Rothfus, Monrad, and Senecal defined a distance parameter in which the velocity was parabolic. They also showed that a corresponding shearing stress could be defined which was linear in the modified distance and which equaled the actual skin friction at the outer wall of the annular space.

The new distance parameters permitted viscous-flow patterns in an annulus to be compared with those in a smooth tube of radius $(r_2^2 - r_m^2)/r_2$, the "equivalent" tube indicated in Equation (11). It was shown that the annular flow corresponded to the flow in the outer portion of the equivalent tube, from the standpoint of velocity distribution. On the assumption that a similar coincidence of local velocities occurred in fully turbulent flow, a modified u^+, y^+ correlation was developed for annuli. Experimental velocity data indicated that annular velocity profiles were in rough agreement with the ordinary u^+, y^+ correlation for tubes, and the effect of Reynolds number on the correlation appeared to be somewhat greater in annuli than in tubes over the lower turbulent range. This was later observed to be true for parallel plates as well, but no attempt was made to resolve the question in the case of annuli.

The velocity correlation for annular conduits has some disturbing features when compared with the parallel-plate correlation previously described. Although the local velocities in the annulus are coincident with those in the equivalent tube, the maximum velocity in the annulus does not occur at the center of the equivalent tube. Thus there is no simple connection between the Reynolds numbers in the annulus and tube such as shown in Equation (7). Furthermore, a pseudo shearing stress, rather than the actual one, is made linear in the modified distance variable; otherwise, the correlating method for annuli is similar to that for parallel plates in that it requires restraints on the tube radius, the skin friction, and the properties of the fluid.

Tubes and parallel plates are effectively the limiting cases of annuli as the radius ratio progresses from zero to unity. It is, therefore, reasonable to assume that a general method of correlating annular velocity profiles might be developed which would include tubes and parallel plates. The foregoing discussion suggests that there is a factor in the behavior of turbulent fluids which has not been accounted for in previous studies of local velocities.

DEVELOPMENT OF VELOCITY CORRELATIONS

The distribution of shearing stress in an annulus can be shown, through a simple force balance, to be

$$\tau g_0 = \frac{\Delta p g_0}{L}\left(\frac{r^2 - r_m^2}{2r}\right) \qquad (12)$$

regardless of the type of flow. It is interesting to note that a hydraulic radius R_H can be defined for the fluid contained between the radius of zero shear r_m and the arbitrary radius r. If the hydraulic radius is defined as the cross-sectional area of the fluid divided by the perimeter of external shear, then

$$\tau g_0 = \frac{\Delta p g_0}{L} R_H \qquad (13)$$

The shearing stress is therefore linear in the hydraulic radius. The same is true, of course, for tubes and parallel plates. Since Equations (12) and (13) can be written with $r = r_2$, the radius of the outer wall, it follows that

$$\frac{\tau g_0}{\tau_2 g_0} = \frac{r_2(r^2 - r_m^2)}{r(r_2^2 - r_m^2)} = \frac{R_H}{R_{H_2}} \qquad (14)$$

The distribution of eddy viscosity, ϵ_a across the annular section is related to the local shearing stress through the definitive equation

$$\tau g_0 = -(\mu + \epsilon_a)\frac{du_a}{dr} \qquad (15)$$

where u_a is the mean local fluid velocity at the point where the shearing stress is measured. Combination of Equations (13), (14), and (15) yields

$$\frac{2(\mu + \epsilon_a)\, du_a}{R_{H_2}\tau_2 g_0} = -\left(\frac{r^2 - r_m^2}{r}\right) dr \qquad (16)$$

If the eddy viscosity proves to be a function of the local fluid velocity, Equation (16) can be integrated from the outer wall to an arbitrary point in the fluid. Thus

$$\frac{2}{R_{H_2}\tau_2 g_0}\int_0^{u_a} (\mu + \epsilon_a)\, du_a$$

$$= \frac{1}{2R_{H_2}^2}\left(r_2^2 - r^2 - 2r_m^2 \ln \frac{r_2}{r}\right) \qquad (17)$$

It is convenient to work in terms of the average value of the total viscosity taken with respect to the local fluid velocity. If the average is designated by the superscript 0, then

$$(\mu + \epsilon_a)^0 u_a = \int_0^{u_a} (\mu + \epsilon_a)\, du_a \qquad (18)$$

In addition, the right-hand side of Equation (17) can be represented by the symbol Φ_0 where the subscript indicates that only the fluid outside the radius of maximum local velocity is being considered. Thus by definition,

$$\Phi_0 = \frac{1}{2R_{H_2}^2}\left(r_2^2 - r^2 - 2r_m^2 \ln \frac{r_2}{r}\right) \qquad (19)$$

Equations (17), (18), and (19) can be combined to yield the basic relationship in the form

$$\frac{2(\mu + \epsilon_a)^0 u_a}{R_{H_2}\tau_2 g_0} = \Phi_0 \qquad (20)$$

By means of a similar development, it can be shown that the last equation applies equally well to the fluid lying inside the radius of maximum velocity provided that the latter retains its viscous-flow value indicated in Equation (9). Knudsen and Katz (3) as well as Rothfus, Monrad, and Senecal have shown experimentally that Equation (9) is valid for fully turbulent as well as fully viscous flow.

If Equation (20) is evaluated at the radius of maximum local fluid velocity r_m, then

$$\frac{2(\mu + \epsilon_a)_m{}^0 u_{ma}}{\Phi_{0m} R_{H_2}\tau_2 g_0} = 1.0 \qquad (21)$$

The maximum value of Φ_0, designated as Φ_{0m}, is obtained from Equation (19) with $r = r_m$. The value of Φ_{0m} varies from 1.00 for parallel plates to 2.00 for tubes. The magnitude of Φ_{0m} is a unique function of the radius ratio r_1/r_2 as shown in Table 1.

It is permissible to write Equation (21) for that particular annulus whose radius ratio is zero, namely the tube. If the radius of the tube is R_0, then

$$R_0 = \Phi_{0m} R_{H_2} \qquad (22)$$

since the hydraulic radius R_{H_2} is simply $R_0/2$. Therefore

$$\frac{2(\mu + \epsilon_p)_m{}^0 u_{mp}}{R_0(\tau_0 g_0)_p} = 1.0 \qquad (23)$$

At this point it is necessary to make a basic postulation about the behavior of the fluid in fully turbulent flow which transcends the geometry of the conduit. A tube and an annulus will be considered for which $R_0 = \Phi_{0m} R_{H_2}$ and $(\tau_0 g_0)_p = \tau_2 g_0$ with the same fluid flowing in both conduits under fully turbulent conditions. These are precisely the same restraints imposed by Rothfus and Monrad on the case of flow between parallel plates. It will be assumed that the eddy viscosity is the same function of the local fluid velocity in both conduits. On the average, therefore, under the stated restraints, Equations (21) and (23) predict that

$$(\mu + \epsilon_a)_m{}^0 u_{ma} = (\mu + \epsilon_p)_m{}^0 u_{mp} \qquad (24)$$

and if the assumption regarding eddy viscosity is correct,

$$u_{ma} = u_{mp} \qquad (25)$$

For flow in a tube of radius R_0, Equation (19) reduces to the form

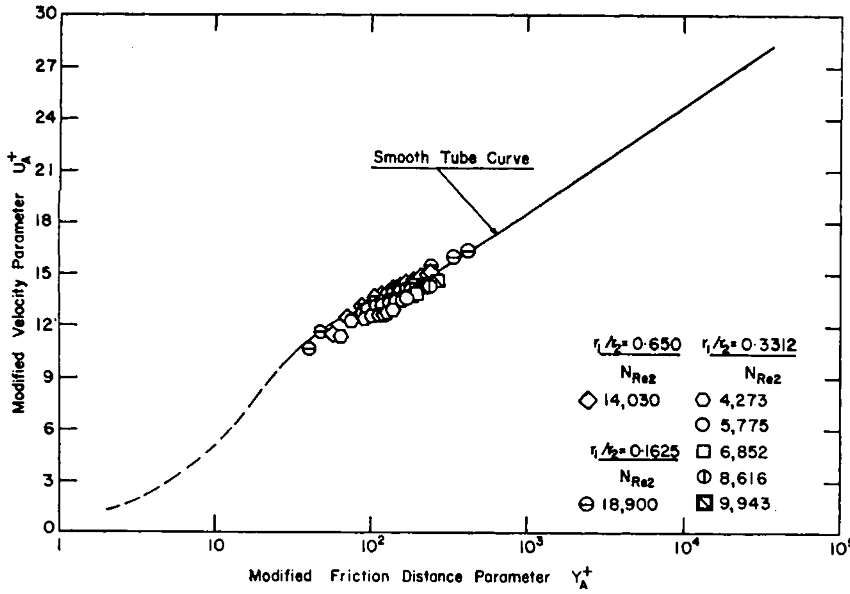

Fig. 1. Correlation of annular velocities inside the radius of maximum fluid velocity.

$$\Phi_0 = 2\left[1 - \left(\frac{R}{R_0}\right)^2\right] \qquad (26)$$

where R is the radial distance from the center of the stream to the point at which the local velocity is measured. If Equation (20) is applied to a tube and an annulus under the restraints previously described and if this assumption about the eddy viscosity is valid,

$$u_a = u_p \qquad (27)$$

provided that the distances R and r are related in a particular way. It is apparent from Equations (20) and (26) that the relationship must be

$$\left(\frac{\Phi_0}{\Phi_{0m}}\right)_a = \left(\frac{\Phi_0}{\Phi_{0m}}\right)_p = 1 - \left(\frac{R}{R_0}\right)^2 \qquad (28)$$

It is customary to express the position of a point within a tube in terms of its distance Y from the wall. Thus, since $Y = R_0 - R$ by definition and $R_0 = \Phi_{0m}R_{H_2}$, Equation (28) can be rewritten in the form

$$Y = (1 - \sqrt{1 - (\Phi/\Phi_{0m})_a})R_{H_2} \qquad (29)$$

Thus if a tube and annulus are operated with the same fluid and with the stated restraints on the linear dimensions and skin frictions, the local velocity at distance r from the center of the annulus should be equal to the velocity in the tube at distance Y from the tube wall.

Since the local velocities, linear distances, and friction velocities for tubes and all other annuli, including parallel plates, have been made equal by the procedure described above, it follows that

$$U_a^+ = U_p^+ = \frac{u_a}{\sqrt{\tau_2 g_0/\rho}}\left(\frac{V}{u_m}\right)_p \qquad (30)$$

and

$$Y_a^+ = Y_p^+ = \frac{[1 - \sqrt{1 - (\Phi_0/\Phi_{0m})_a}]\Phi_{0m}R_{H_2}\sqrt{\tau_2 g_0/\rho}}{\nu(V/u_m)_p} \qquad (31)$$

by virtue of the definitive Equations (1) and (2). The unique U^+, Y^+ correlation for smooth tubes can, therefore, be expected to represent velocity distributions in annuli and parallel plates.

The Reynolds number in the equivalent tube, defined in the manner of Equation (6), must be

$$N_{Re_p} = \frac{2R_0V_p}{\nu} = \frac{2\Phi_{0m}R_{H_2}V_p}{\nu} \qquad (32)$$

The Reynolds number for the annulus can be defined through Equation (11), that is,

$$N_{Re_2} = \frac{4R_{H_2}V_a}{\nu} \qquad (11a)$$

Therefore, since the maximum local velocities in the tube and annulus are equal, as shown in Equation (25),

$$N_{Re_p} = \frac{\Phi_{0m}}{2}N_{Re_2}\frac{(V/u_m)_p}{(V/u_m)_a} \qquad (33)$$

In like fashion, since the skin frictions and fluid densities are equal,

$$\sqrt{\frac{f_p}{f_2}} = \frac{(V/u_m)_a}{(V/u_m)_p} \qquad (34)$$

It is apparent that since Φ_{0m} is unity for flow between parallel plates and $f_2 = f_P$ by definition, Equations (33) and (34) are exactly equivalent to Equations (7) and (8). The proposed method of correlating local velocities in annuli is consistent in every way with the parallel-plate method suggested by Rothfus and Monrad. The assumption regarding eddy viscosities can be verified for the case of annuli by means only of experimental velocity data.

EXPERIMENTAL VELOCITY MEASUREMENTS

Mean local fluid velocities were measured in a smooth, concentric annulus and in a smooth, rectangular duct of large aspect ratio. The outer boundary of the annulus was a Lucite tube with an inner diameter of 0.750 in. The core was a smooth copper tube with an outer diameter of 0.248 in. The radius ratio was, therefore, 0.331. The rectangular duct, formed from smooth brass plates, was 14 in. wide with a clearance of 0.700 in. between its broad surfaces. Adequate calming lengths were provided in each case to ensure a fully developed velocity profile at the point of measurement.

The test fluid was water at room temperature. The liquid was recirculated through heat exchangers to maintain isothermal, steady flow. Local velocities were measured by means of calibrated impact tubes formed from hypodermic needle tubing. The impact pressures were indicated on vertical U-tube manometers. In order to obtain the necessary degree of multiplication in the manometer readings, monochlorobenzene and monofluorobenzene were used as the manometer fluids. Since the specific gravities of monochlorobenzene and monofluorobenzene at 20°C. are only 1.1084 and 1.0267, respectively referred to water at 20°C., the desired amplification could be obtained without resorting to the use of micromanometers. Special precautions were observed in the design and operation of the manometers to prevent contamination of the liquid-liquid interfaces.

Friction data as well as velocity measurements were obtained over the viscous, transition, and lower turbulent ranges of flow. Since transition behavior is a separate subject worthy of individual treatment, only the data obtained in the fully turbulent-flow regime are reported herein. It should be recognized, however, that the lower limit of full turbulence on the Reynolds-number scale has been established through velocity measurements.

RESULTS AND DISCUSSION

The experimental velocity data are presented in Figures 1, 2, and 3 on modified u^+, y^+ coordinates. In each case comparison is made with the best line for smooth tubes based on the velocity data of Nikuradse (*5*). Diessler (*2*), Rothfus, Monrad, and Senecal (*9*), and Senecal and Rothfus (*10*). The data for the annulus have been supplemented by the experimental results of Rothfus, Monrad, and Senecal in order to extend the radius ratio range.

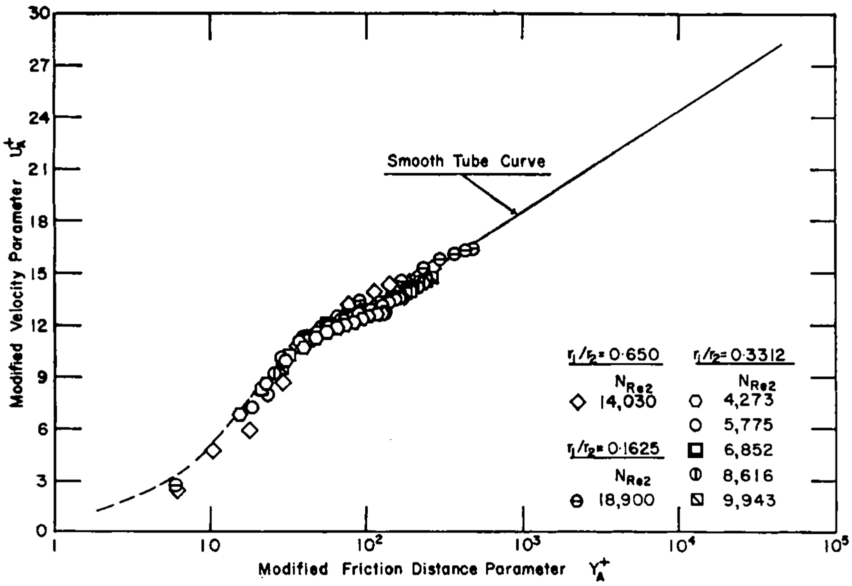

Fig. 2. Correlation of annular velocities outside the radius of maximum fluid velocity.

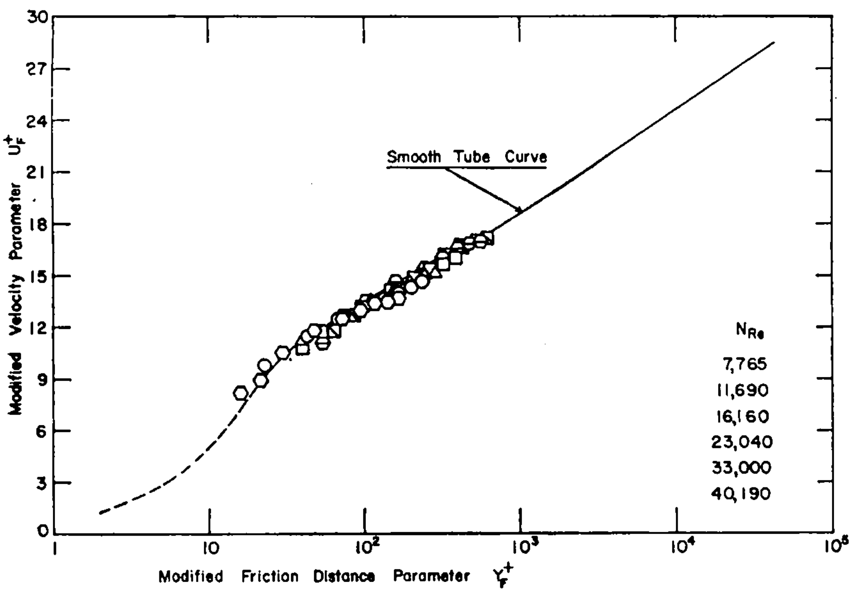

Fig. 3. Correlation of velocities for parallel plates.

Table 1. Effect of Radius Ratio on the Parameter Φ_{0m}

r_1/r_2	Φ_{0m}
0.00 (tubes)	2.000
0.10	1.475
0.20	1.386
0.40	1.257
0.60	1.134
0.80	1.062
1.00 (parallel plates)	1.000

It is apparent that excellent agreement with the smooth-tube data is obtained through the proposed method of correlation. The tube relationship can be used with equal precision to predict velocity profiles in annuli and parallel plates. In the case of parallel plates, the data of Sage and coworkers (*1, 6, 7*) verify the results of the present work but are too numerous to include in Figure 3. The excellent agreement between these data and the smooth-tube curve has already been shown by Rothfus and Monrad (*8*).

Figure 4 is included to aid the reader in visualizing the form of fully turbulent velocity distributions in the experimental annulus. It is apparent that the point of maximum local velocity lies closer to the core than to the outer wall. The actual value of the radius of maximum velocity is in almost exact coincidence with the viscous-flow value predicted in Equation (9). The mutual agreement of Figures 1 and 2 also confirms this observation since Equation (20) can be applied on both sides of the maximum point only when Equation (9) is obeyed.

In order to calculate local velocities in the proposed manner, values of $(V/u_m)_p$ must be obtained for use in Equations (30) and (31). Since the velocity ratio is readily available as a function of the Reynolds number in the equivalent tube, the problem is one of evaluating the latter quantity. Ordinarily, the Reynolds number for the annulus in question is known or can be estimated tentatively. If the velocity ratio $(V/u_m)_a$ is known at N_{Re_2}, the Reynolds number N_{Re_p} and the velocity ratio $(V/u_m)_p$ can be obtained from Equation (33) by means of trial and error.

Unfortunately, too few data on annuli are available to permit prediction of $(V/u_m)_a$ at various radius ratios and Reynolds numbers from experimental information. On the other hand, the authors have measured fluid friction in various annuli and between parallel plates. As a result, the friction factor f_2 at the outer wall is available over a wide range of radius ratios in the lower turbulent range of flow. By a fortuitous circumstance, this proves to be sufficient information from which to calculate $(V/u_m)_a$ without much difficulty.

It is found that the friction factors for all annuli, including tubes and parallel plates, are the same function of the Reynolds number N_{Re_2} in the fully turbulent range. This fact, first observed by Rothfus, Monrad, and Senecal, is illustrated in Figure 5 by the data of the present authors. Since the friction factors are unique functions of the Reynolds number, the ratio $(V/u_m)_a/(V/u_m)_p$ depends on N_{Re_p} and N_{Re_2} only, by virtue of Equation (34). Thus Equation (33) can be written in the general form

$$(V/u_m)_a = \psi(N_{Re_2}, \Phi_{0m}) \qquad (35)$$

The necessary trial-and-error calculations have been performed and the solution of the last equation is shown graphically in Figure 6. Once the annular dimensions and operating conditions are fixed, the ratio of average to maximum velocity $(V/u_m)_a$ can be obtained directly from the graph at the appropriate value of N_{Re_2} and Φ_{0m}. The Reynolds number N_{Re_p} for the equivalent tube and the velocity ratio $(V/u_m)_p$ at that Reynolds number can then be obtained from Equation (33) and Figure 6 by trial and error. The friction velocity $(\tau_2 g_0/\rho)^{1/2}$ is obtainable by means of Equation (10) and Figure 5. The velocity distribution in the annulus can, therefore, be determined from the smooth-tube curve of U^+ against Y^+ shown in Figure 1, 2, or 3.

It should be noted that coincidence of local velocities under the stated restraints does not necessarily result in a criterion for the stability of the turbulent regime in various types of ducts. The U^+, Y^+ correlation for smooth tubes is independent of Reynolds number in only the fully turbulent range above 3,000. When applied to annuli and parallel plates, the correlation cannot, therefore, be unique at Reynolds numbers below the value corresponding to a Reynolds number of 3,000 in the equivalent tube. The Reynolds number N_{Re_2} marking the lower limit of the correlation must consequently depend on the radius ratio in accordance with Equation (33). On the other hand, the friction data indicate that full turbulence in the noncircular ducts is maintained until somewhat lower Reynolds numbers are reached. No attempt has been made to evolve a criterion for the critical Reynolds number.

In summary, a consistent and effective method is now available for calculating local velocities in tubes, concentric annuli, and parallel flat plates. Friction data provide a basis for determining intermediate quantities necessary to the calculation of velocities. Although the number of supporting data is limited, there is ample reason to conclude that the recommended correlation can be used in the fully turbulent flow regime over the entire range of radius ratios.

NOTATION

b = half clearance between parallel plates, ft.
f = Fanning friction factor, dimensionless
g_0 = conversion factor = 32.2 (lb.-mass)(ft.)/(lb.-force)(sec.^{2})
L = length of conduit, ft.
N_{Re} = Reynolds number, dimensionless
N_{Re_2} = Reynolds number defined in Equation (11), dimensionless
Δp = pressure drop due to friction, lb.-force/sq. ft.
r = radial distance from center of configuration to a point in the fluid stream, ft.; r_0 = radius of tube, ft.; r_1 and r_2 = inner and outer radii of an annulus, respectively, ft.
r_m = radius of maximum local fluid velocity, ft.
R = radial distance from center of equivalent tube, ft.; R_0 = radius of equivalent tube, ft.
R_H = hydraulic radius for the fluid between r_m and r, ft.; R_{H_2} = hydraulic radius for the fluid between r_m and r_2, ft.
u = mean local fluid velocity, ft./sec.
u_m = maximum value of mean local fluid velocity, ft./sec.
u^+ = velocity parameter = $u/\sqrt{\tau_0 g_0/\rho}$, dimensionless
U^+ = modified velocity parameter defined in Equation (1), dimensionless
V = bulk average linear velocity of fluid, ft./sec.
y = distance from wall to a point in the fluid stream, ft.
y^+ = friction distance parameter = $y\sqrt{\tau_0 g_0/\rho}/\nu$, dimensionless
Y = distance from wall of equivalent tube, ft.
Y^+ = modified friction distance parameter defined in Equation (2), dimensionless

Greek Letters

ϵ = eddy viscosity, lb.-mass/(sec.)(ft.)
μ = viscosity of fluid, lb.-mass (sec.)(ft.)
ν = kinematic viscosity of fluid, sq. ft./sec.

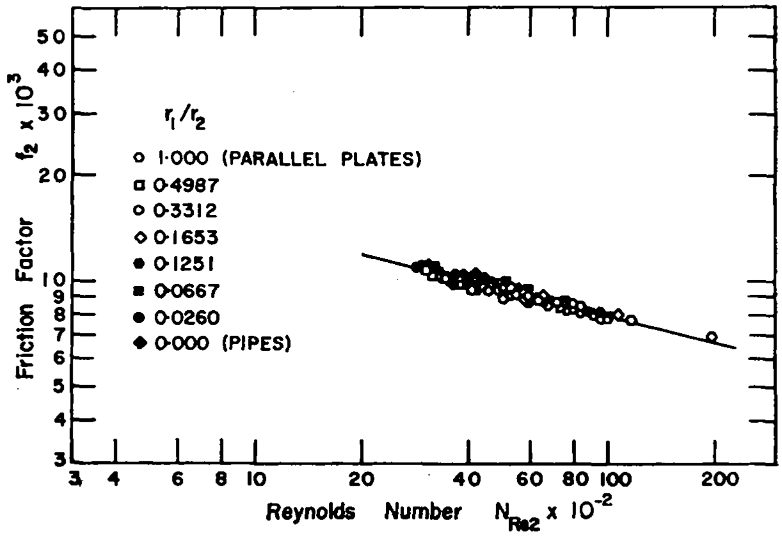

Fig. 5. Correlation of fanning friction Factors at the outer surface of annuli.

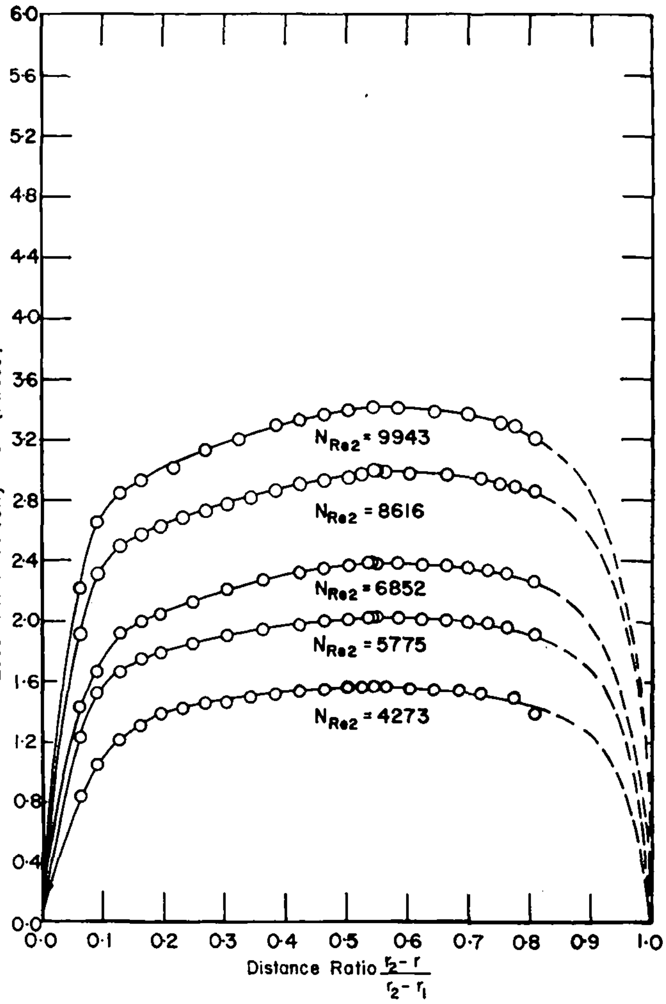

Fig. 4. Velocity profiles in the annulus of radius ratio = 0.3312.

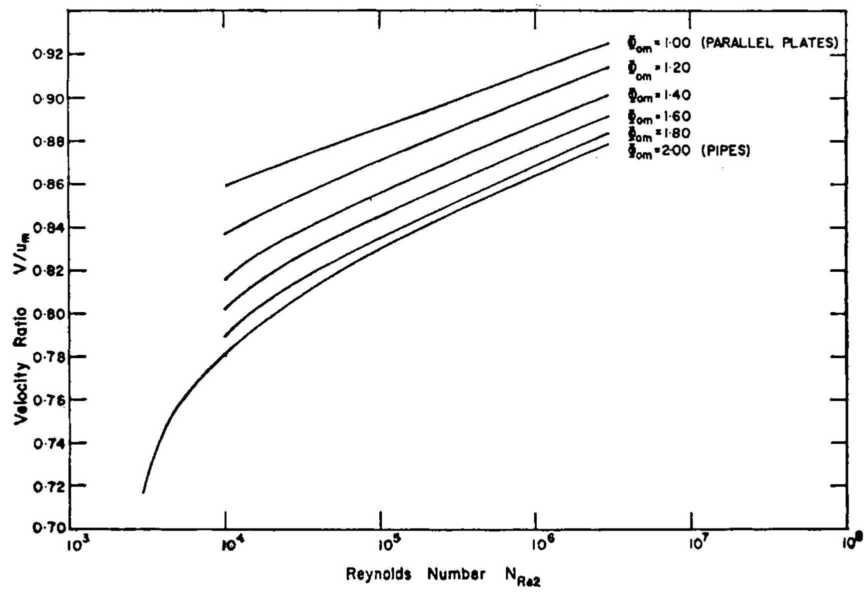

Fig. 6. Ratio of average to maximum velocity in tubes and non circular conduits.

ρ = density of fluid, lb.-mass/cu. ft.

τ = local shearing stress in fluid, lb.-force/sq. ft.; τ_0 = skin friction at wall of conduit, lb.-force/sq. ft.; τ_2 = skin friction at outer wall of annulus, lb.-force/sq. ft.

Φ_0 = geometrical function defined in Equation (19), dimensionless; Φ_{0m} = maximum value of geometrical function, dimensionless

ψ = function in Equation (35), dimensionless

Subscripts

F = parallel flat plates

p = pipes or tubes

a = annuli

m = association with the point of maximum fluid velocity

Superscript

0 = average taken with respect to the mean local fluid velocity

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Liquid-liquid Extraction Accompanied by Chemical Reaction

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From time to time, A.I.Ch.E. Journal is going to present translations of certain technical articles written by our Japanese colleagues in their own language. These translations are to be made by Kenzi Etani, who received his B.S. in chemical engineering in 1953 at the Tokyo Institute of Technology and his M.S. in 1955 at M.I.T. He is associated with Stone & Webster and is an associate member of American Institute of Chemical Engineers. He is also a member of the Society of Chemical Engineers, Japan, and the Japan Oil Chemists' Society. His offer to help break down the language barrier is acknowledged.

The following article (two will follow in the September and December issues, respectively) was published in *Chemical Engineering (Japan)*, volume 21, pages 75–79 (1957).

Abstracts, notation, literature cited, tables, and figure captions not published here appear in English in the original paper. No figures will be reproduced in these translations.

Liquid-liquid extraction accompanied by chemical reaction was studied with the use of the following solutes in water and benzene with known interfacial area:

Upper layer Solvent	Solute	Lower layer Solvent	Solute	Transferred material
benzene	iodine	water	sodium hyposulfite	iodine
benzene	iodine	water	sodium hyposulfite + NaI	iodine
benzene	benzoic acid	water	KOH or NaOH	benzoic acid
benzene	monochloro-acetic acid	water	KOH or NaOH	monochloro-acetic acid
benzene	butyric acid	water	KOH or NaOH	butyric acid

EXPERIMENTAL PROCESS

Apparatus used in this experiment is shown in Figure 1. Two known concentration liquids were introduced into the apparatus, then stirred by two mixers, with no break occurring in the surface between the two liquid layers. The stirring speeds of benzene and water were 71 and 78.5 rev./min. respectively, with rotation in the same direction. Benzene was sampled intermittently from the container to determine concentration change. The solute concentration in water was calculated by material balance.

Extraction of I_2 in Benzene by Sodium Hyposulfite Solution

The relation between the rate of extraction and sodium hyposulfite concentration at three I_2 concentrations in benzene is shown in Figure 3. All white points in this figure indicate experimental data (black points will be mentioned later.) At a low concentration of sodium hyposulfite, the rate of extraction increases linearly with the increase of

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